INFINITY

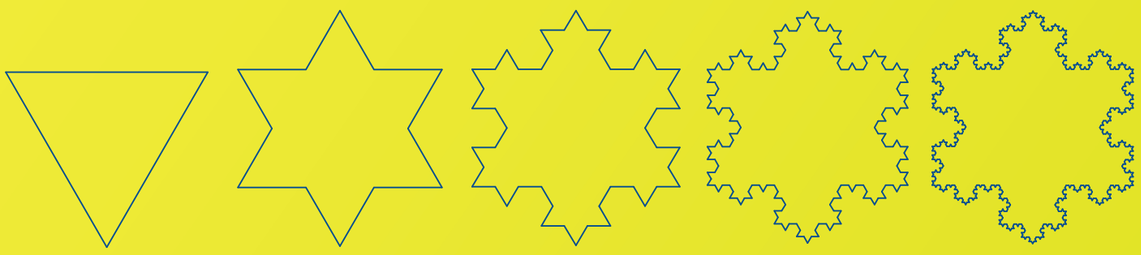
In Small Spaces

Infinity can be found in the strangest places—as demonstrated by three mathematicians: Koch, Sierpinski and Menger. They found that by starting with basic shapes and applying simple sets of rules repeatedly, they could generate shapes and solids with quite strange properties. Koch's Snowflake, Sierpinski's Triangle and Menger's Sponge are famous examples of the limits that mathematics can reach.

Koch's Snowflake

- Start with an equilateral triangle.
- Divide each side into three equal pieces.
- Replace the middle third of each side with the other two sides of an equilateral triangle.
- Repeat this process using the new sides. Repeat indefinitely.

The first five steps of this process are shown below.

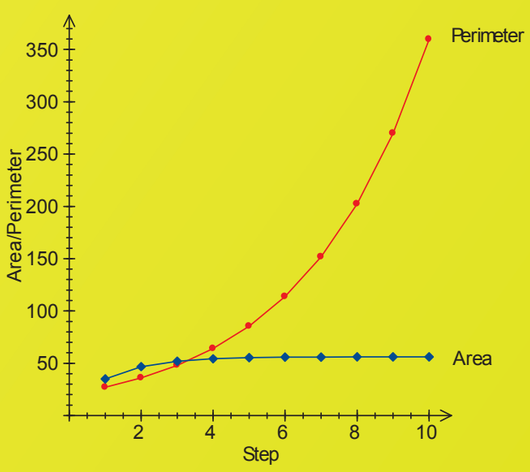


What Happens to the Perimeter?

Every time we add a triangle to a side, we increase the perimeter. Where we once had three units of length - we now have four. This means that the perimeter is increased by one third every step. It is growing exponentially!

What Happens to the Area?

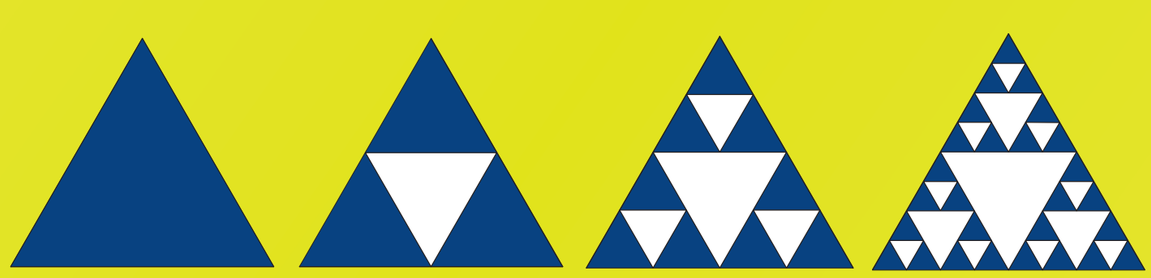
Every step, the number of triangles we have to add (3, 12, 48, 192…) multiplies by four but the size of each triangle is only one ninth of the size of the previous steps. The amount of area we add at each step gets smaller and smaller.



After 95 steps, a Koch snowflake based on a 9cm triangle has an area of just over 56.1184 cm^2 and a perimeter of just under 150 million kilometres. That is the distance from the sun to the earth. After 134 steps, the perimeter is greater than the diameter of our galaxy! If you repeat the process indefinitely, the perimeter becomes infinite but the area is still only just over 56.1184 cm^2.

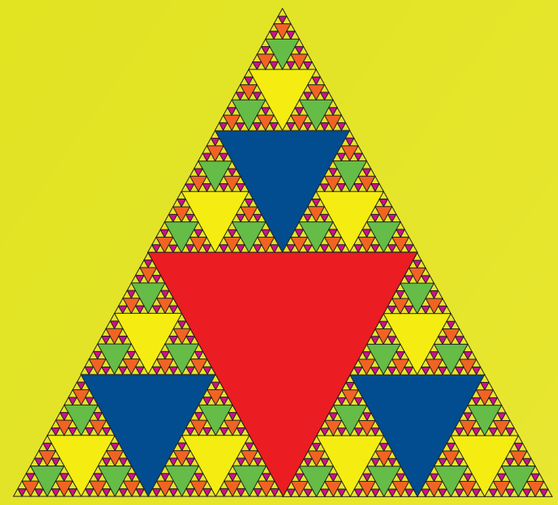
Sierpinksi's Triangle

Sierpinski's triangle is built by removing the central triangle from an equilateral triangle and then repeating this process with the remaining triangles. The first four steps are shown below. If continued indefinitely the shape becomes an infinitely complex mesh that has an area of zero.



This step seven Sierpinski triangle has had the triangles removed at each step coloured a different colour.

Can you calculate the proportion of the triangle that remains after each of the first seven steps?

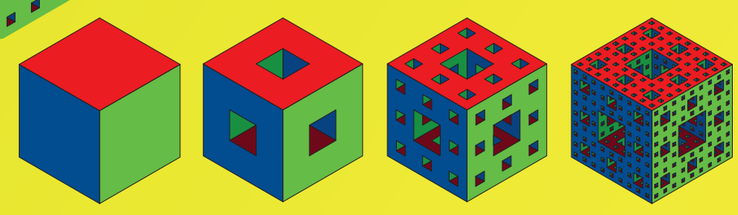


Menger's Sponge

Menger's sponge is the 3D cousin of Sierpinski's triangle. Start with a 3 by 3 cube and remove the centre from each side. The holes pass all the way through the cube. Repeat this process as shown in the diagram below. The large diagram is a step 5 Menger sponge.

If you continue this process indefinitely, the sponge has an infinite surface area but a volume of zero.

Can you calculate how much of the cube remains after each step?



Niels Fabian Helge von Koch (1870-1924) was a Swedish mathematician who developed the "Koch Snowflake" in 1906. The Koch snowflake is a continuous curve that is infinitely long, encloses a finite area, but does not have a tangent at any point.

Waclaw Sierpinski (1882-1969) spent most of his life in Poland and experienced occupation by both the Russian and German armies. He produced outstanding mathematics during his whole career despite some horrendous experiences.

Karl Menger (1902-1985) was born in Austria but moved to the USA just before World War II. He was instrumental in the development of American mathematics in the twentieth century.